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Journal of Sound and Vibration 298 (2006) 453–461
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

Sensitivity analysis of free vibration characteristics of an in situ railway concrete sleeper to variations of rail pad parameters

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Received 28 April 2006; received in revised form 24 May 2006; accepted 27 May 2006
Available online 8 August 2006

Abstract

The vibration of in situ concrete sleepers in a railway track structure is a major factor causing cracking of prestressed concrete sleepers and excessive railway track maintenance cost. Not only does the ballast interact with the sleepers, but the rail pads also take part in affecting their free vibration characteristics. This paper presents a sensitivity analysis of free vibration behaviors of an in situ railway concrete sleeper (standard gauge sleeper), incorporating sleeper/ballast interaction, subjected to the variations of rail pad properties. Through finite element analysis, Timoshenko-beam and spring elements were used in the in situ railway concrete sleeper modeling. This model highlights the influence of rail pad parameters on the free vibration characteristics of in situ sleepers. In addition, information on the first five flexural vibration modes indicates the dynamic performance of railway track when using different types of rail pads, as it plays a vital role in the cracking deterioration of concrete sleepers.

1. Introduction

Due to the nature of undergoing numerous dynamic loadings on railway track, the vibration characteristics of in situ railway concrete sleepers are essential in analysis and design procedures. Clearly, the sleeper damage occurs mostly at resonant frequencies of the sleepers, especially for the dominance in the first five modes of vibration. There have been a number of studies shown that the resonant vibrations of sleepers affect not only the sleepers themselves, but also the wheel–rail interaction forces [1–3]. Grassie and Cox [2] presented the responses of railway track to high-frequency excitation. They also found that the higher pad stiffness tends to increase the contact forces. It is very important to note that the typical corrugation-passing frequencies (e.g. 750 Hz for a corrugation of 60 mm wavelength and train speed of 45 m/s) are most likely to cause much damage on concrete sleepers at their second and third dynamic mode shapes. Esveld [4] also described that the normal train operation causes the dynamic forces at the frequency range from 0 to 20 Hz for sprung mass and from 20 to 125 for unsprung mass. In particular, corrugations, bad welds, and wheel flats would create the forces varying from 0 to 2000 Hz, depending on the train speeds. An analytical model for analyzing dynamic

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.05.034

behaviors of concrete sleepers in both free–free and in situ conditions was later developed by Dahlberg and Nielsen [5]. Subsequently, based on a number of experimental results, Grassie [6] developed a simplified two-dimensional dynamic modeling for vibration analysis of concrete sleepers in free–free condition. It has been found from the last two works that the Timoshenko beam element is the best approximation of the concrete sleepers, even though the elastic properties of prestressed concrete sleepers may not be exact. Those analytical models were treated through sophisticated numerical procedures, e.g. the modified Wittrick–Williams bisection algorithm. Ilias [7] investigated the effect of rail pad stiffness on wheelset/track interaction and corrugation growth. It was found that the stiffer the rail pad, the higher the global track resonance. However, it has been found that the effect of rail pad parameters on the dynamic responses of in situ railway concrete sleepers has not been studied, although the modern rail pads play a vital role on the track dynamics nowadays [8,9]. In practice, the rail pads are replaced with new synthetic ones during the track maintenance, while their influence on dynamic changes within the concrete sleepers themselves is yet unknown. The structural behaviors of the railway concrete sleepers in the track structure system or so-called 'the in situ railway concrete sleepers' can be found in a recent work [10].

This paper presents the results of a sensitivity analysis of free vibration characteristics of the in situ railway concrete sleepers to variations of rail pad properties. The concrete sleeper modelled herein is the modified Australian standard gauge sleeper type, whose similar results (Swedish sleeper) can be found in literature [5,6]. The emphasis was placed on only the in situ mono-block sleepers fitted in ballasted railway tracks. The aim was to examine the effect of rail pad parameters on the dynamic behavior of in situ sleepers while the sleeper/ballast interaction was also taken into account. The in situ sleepers were considered as beams on elastic foundation of the Winker type. The in situ sleeper model was analyzed based on the finite elements using a computer package, STRAND7. Two-dimensional beam element, considering shearing effects, was employed as the concrete sleeper to embrace the shear deformation and rotational inertia, whereas the ballast-support system and rail pad were modelled using elastic support feature and spring elements, respectively. Changes in natural frequencies and dynamic mode shapes of the sleepers are presented to describe the effect of rail pads with various types. Information on dynamic changes is of significant benefit to the research on non-destructive testing and health monitoring of on-track concrete sleepers.

2. In situ railway concrete sleeper modeling

Considering the system of an in situ sleeper model in Fig. 1, the Timoshenko beam elements were adopted for modeling the sleeper, in order to obtain better agreement at higher frequencies because the rotatory inertia

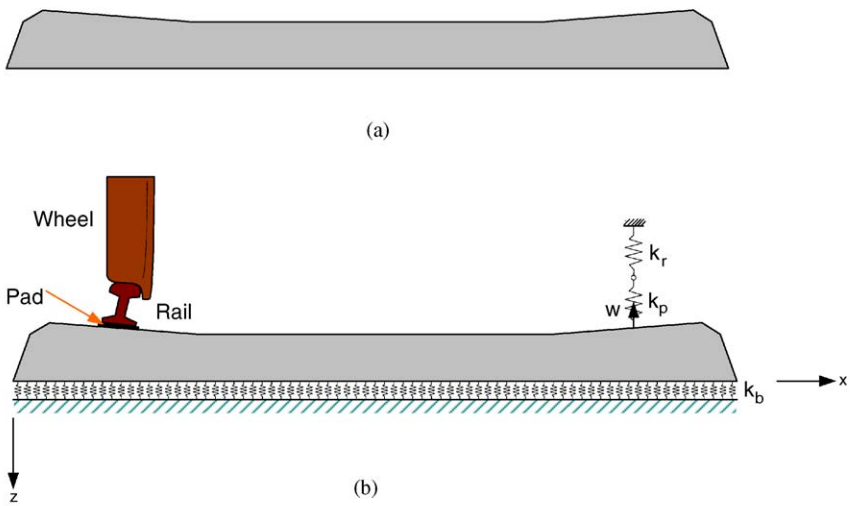


Fig. 1. Typical models of railway concrete sleepers: (a) free-free condition; (b) in situ condition.

and shear deformation have been taken into account in the dynamic simulation. The free–free condition exists when the sleeper is virtually in space without any constraints, whilst the in situ one represents the condition of assembled sleepers resting on ballast in the actual track. The equations of motion for free vibrations of the in situ sleeper system can be written as follows [11–13]:

$$\frac{\partial}{\partial x}\kappa AG\left[\psi(x,t)-\frac{\partial z(x,t)}{\partial x}\right]+m_s\frac{\partial^2 z(x,t)}{\partial x^2}+c_b\frac{\partial z(x,t)}{\partial t}+k_b z(x,t)=\vec{F}(x,t), \quad (1)$$

$$\frac{\partial}{\partial x}EI\left[\frac{\partial \psi(x,t)}{\partial x}\right]-\kappa AG\left[\psi(x,t)-\frac{\partial z(x,t)}{\partial x}\right]-m_s r_s^2\frac{\partial^2 \psi(x,t)}{\partial x^2}=0, \quad (2)$$

where

$$\vec{F}(x,t)=\sum_{i=1}^{2}\{k_{pi}[w(x_i,t)-z(x_i,t)]+c_{pi}[\dot{w}(x_i,t)-\dot{z}(x_i,t)]\}\,\delta x_i. \quad (3)$$

Herein k_b and c_b stand for the stiffness and damping constant of ballast support system; k_p and c_p stand for the stiffness and damping constant of rail pads; $z(x, t)$ is the vertical deflection of sleeper; $\psi(x, t)$ is the rotation angle of sleeper about neutral axis; EI is the effective sleeper flexural rigidity; κGA is the effective sleeper shear distortion rigidity; m_s is the sleeper mass per unit length; and, r_s is the radius of gyration of sleeper cross-section.

In the case of non-uniform sleepers, the effective moment of inertia has been adopted [6]. This proposed methodology is widely accepted since it is simple and convenient.

$$EI=\sqrt{(EI_r)(EI_c)}, \quad \kappa GA=\sqrt{(\kappa GA_r)(\kappa GA_c)}, \quad (4a,b)$$

where EI_c and EI_r are sleeper flexural rigidity at center, and at rail seats, respectively; κGA_c and κGA_r are the sleeper shear distortion rigidity at center and at rail seats, respectively. The analytical solutions and the comparison with the finite element method (FEM) results of the in situ concrete sleeper system are provided elsewhere [5]. Wittrick–Williams bisection algorithm for simplifying eigenvalue problems was employed for the analytical solutions, which are exact for the particular cases [13].

In this simplified FE model, the effective stiffness (k_e), computed from spring series representing rail and rail pad stiffness (k_r and k_p, respectively), was employed as follows:

$$\frac{1}{k_e}=\frac{1}{k_r}+\frac{1}{k_p}. \quad (5)$$

3. Sensitivity analysis

Free vibration characteristics of concrete sleepers with several types of rail pads assembled in the in situ sleeper model were investigated using two-dimensional finite element modeling. The pad stiffness used in this study was varied from 0 to 5000 MN/m. These stiffness values cover the range of all modern rail pads, from very soft rubber, to polymeric, to high-density-polyethelene, or EVA pads (see Table 1). In contrast, the rail stiffness is usually limited depending on different rail sections and their material properties. In standard gauge tracks of UK, the rail stiffness (effective vertical rail stiffness per wheel) is about 62 MN/m [14,15], and however, the rail stiffness of about 72–80 MN/m was also adopted in Cai's model [12]. Note that the rail stiffness in general free vibration analyses can be estimated from the relationship of either finite/virtual-work deformation of the rail at sleeper position or wheel/rail elastic contact function at the similar point. In this study, the maximum rail stiffness at 100 MN/m is employed to generate the effective stiffness (taken rail pad and fastener into account) from a wide range of rail pad parameters. The sleeper was modelled using 50 beam elements with trapezoidal cross-section, while the rail pads and the ballast supporting system were modelled using a spring-damper element and the elastic beam support feature found in STRAND7 FE package [16,17]. The properties of the concrete sleepers studied were obtained from recent experiments [9,18], partially identical to previous studies [5,6], as shown in Table 2. The results of natural frequency analyses for the selected concrete sleepers in both free–free and in situ conditions are tabulated in Table 3. The results are in reasonable agreement with the previously published data [5,6]. It can be noticed from the results that the maximum difference between the experimental and numerical results is less than 7%, and the difference between analytical and numerical solutions is less than 10%. Moreover, the increment of frequencies between free–free and in situ conditions seems to have excellent behavioral trend between previous analytical results and current FEM solutions.

The results of natural frequencies and mode shapes of the in situ railway concrete sleeper are presented in Tables 4 and 5 for the different rail pad properties. Table 4 presents the influence of effective parameters on the natural frequencies of the in situ railway concrete sleeper in the track system. It should be noted that these results are focused on the sleeper behaviors due to a development in health monitoring of track components that one often measures the dynamic behaviors of on-track sleepers by installing accelerometers on sleeper surface [18]. It is found previously from Eq. (5) that the pad parameters have a nonlinear, significant contribution to effective parameters. This results in the dramatic influence on the first three modes of vibration

Table 1
Dynamic stiffness of commercial railway pads[a]

Type	Stiffness (MN/m)	Visual identification
Rubber	20–100	Soft
Studded polymer	200–800	Soft
Polyurethane	800–1200	Medium
High density polyethylene (HDPE)	800–2500	Hard
EVA	3000–3500	Hard
Steel	5000+	Very stiff

[a]Adopted from [4,8,12].

Table 2
Properties used in concrete sleeper validation model

Parameter lists		
Flexural rigidity	$EI_c = 4.60$, $EI_r = 6.41$	MN/m^2
Shear rigidity	$\kappa GA_c = 502$, $\kappa GA_r = 628$	MN
Ballast stiffness	$k_b = 13$	MN/m^2
Effective stiffness	$k_e = 17$	MN/m
Sleeper density	$\rho_s = 2750$	kg/m^3

Table 3
Natural frequencies of the validated concrete sleeper (Hz)

Mode no.	Natural frequencies in free–free condition (Hz)				Natural frequencies in in situ condition (Hz)			
	[4] (Experimental)	[4] (Analytical)	[5] (Analytical)	This study (FEM)	[4]		This study	
					(Analytical)	% increase	(FEM)	% increase
1	113	118	126	112	131	11.0	122	9.0
2	335	327	338	308	335	2.4	314	2.0
3	639	624	637	602	631	1.0	607	1.0
4	995	986	1004	994	989	0.3	997	0.3
5	1390	1391	1423	1484	1393	0.1	1486	0.1

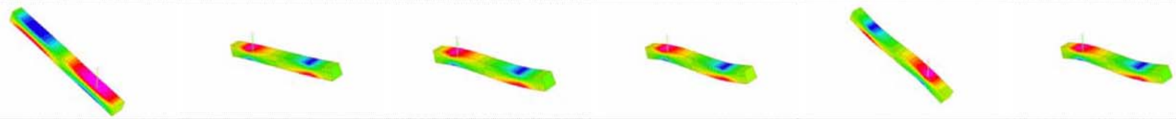
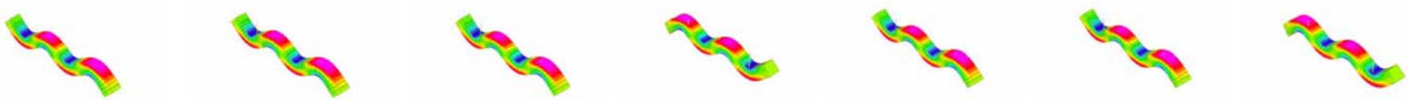
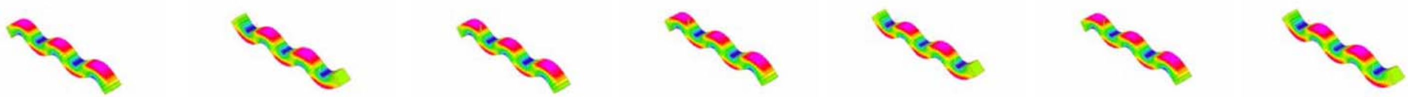
Table 4
Natural frequencies of the concrete sleeper under various pad parameters

Pad stiffness (MN/m)	Effective stiffness (MN/m)	Frequencies (Hz)						
		Rigid body motion		Flexural vibrations				
		Translation	Rotation	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
0	0	48.29	48.27	121.57	311.14	604.04	995.63	1485.11
20	17	68.66	70.59	122.21	313.45	607.30	997.47	1485.49
33	25	76.08	78.70	122.61	314.55	608.84	998.34	1485.67
100	50	94.16	99.16	124.64	318.15	613.69	1001.08	1486.23
200	67	102.85	110.35	127.22	320.71	617.00	1002.95	1486.61
300	75	105.95	115.09	128.94	321.95	618.57	1003.84	1486.79
500	83	108.44	119.54	131.04	323.20	620.14	1004.72	1486.97
800	89	109.95	122.71	132.83	324.15	621.32	1005.39	1487.11
1000	91	110.38	123.74	133.46	324.47	621.71	1005.61	1487.16
1500	94	110.99	125.25	134.44	324.95	622.30	1005.95	1487.22
2500	96	111.34	126.24	135.12	325.28	622.70	1006.17	1487.27
3500	97	111.53	126.73	135.46	325.44	622.89	1006.28	1487.29
5000	98	111.70	127.22	135.80	325.60	623.09	1006.39	1487.32
∞	100	112.03	128.19	136.50	325.92	623.49	1006.62	1487.36

(translation, rotation, and the first flexural modes), as illustrated in Fig. 2, at low to moderate stiffness up to 800 MN/m (e.g. rubber, studded polymeric, or soft HDPE pads). Then, the higher pad stiffness has slight effect on the effective parameters and the consequent natural frequencies of those modes as the almost horizontal line prevails in Fig. 2. The mode frequencies shift up to 2 times for translation and for rotation mode, as well as to 12% for the first bending mode. Fig. 3 presents the normalized modal change in terms of frequency of the in situ sleeper under various rail pad parameters. Interestingly, it is worth noting from Fig. 3 that the lowest frequency or translation mode of vibration has lesser degree of consequential alteration than the rotation mode. For modes 3 and 4, their frequencies are likely to be slightly affected by the rail pad stiffness about 1–5% change in frequencies. In contrast, the fifth-mode frequencies seem to be relatively insensitive to increases in either rail pad or fastener stiffness (less than 1% change in frequencies). However, Table 4 notices the tendency for further rise of the natural frequencies of the all bending modes of vibration if the effective stiffness is escalating beyond 100 MN/m, whilst the rigid-body-motion modes remain constant.

Table 5 delineates the changing period of the transfiguration of vibration mode shapes of the in situ railway concrete sleepers in the track system when using the variety of rail pads. The transfiguration is the behavior that dynamic mode shapes change with parameters or amplitudes of vibration. It should be noted that in the translation and rotation rigid body modes, there exist gradual curvatures behaving like the first symmetrical bending modes and the first anti-symmetrical bending mode when the pad parameters approach 200 MN/m (when using studded pads). In these cases, although the movements are attributed to rail-seat springs and the whole mass of the sleeper moves like the rigid-body translation or rotation, the sleeper elements themselves tend to deform in flexures. At very high pad parameters, these modes closer resemble those bending shapes of sleepers. It can be explained that once the spring elements at rail seat relatively no longer move, the end constraints are imposed inherently and then the rigid body movement would not exist. As a result, the transverse vibrations would rather dominate analogously as the manners of continuum solids or multi-degree-of-freedom systems. These behaviors could also explain why the on-track concrete sleepers sometimes behave similarly to bending while the corresponding frequencies are lower than those in isolated conditions (free–free boundary condition) in previous experiments [19]. It is also observable that curvatures of the flexural mode shapes gradually alter during the increases of rail pad stiffness. Numerical sensitivity analyses have been performed to further investigate the behaviors of the in situ concrete sleeper for the infinite effective stiffness. Fig. 4 illustrates the free vibration behaviors of fully restrained condition (when $k_p = \infty$). These behaviors can occur when the supports alter the movement of the in situ sleeper at both rail seats, resulting in distorting the

Table 5
Changes in mode shapes of the concrete sleeper under various rail pad parameters

Mode	Pad parameters, k_p						
	Free–Free condition	Without pad (Beam on foundation)	Rubber pads, 20 MN/m	Studded pads, 200 MN/m	HDPE pads, 1500 MN/m	EVA pads, 3500 MN/m	Steel plate, 5000 MN/m
Translation (rigid body)	-						
Rotation (rigid body)	-						
1st bending mode							
2nd bending mode							
3rd bending mode							
4th bending mode							
5th bending mode							

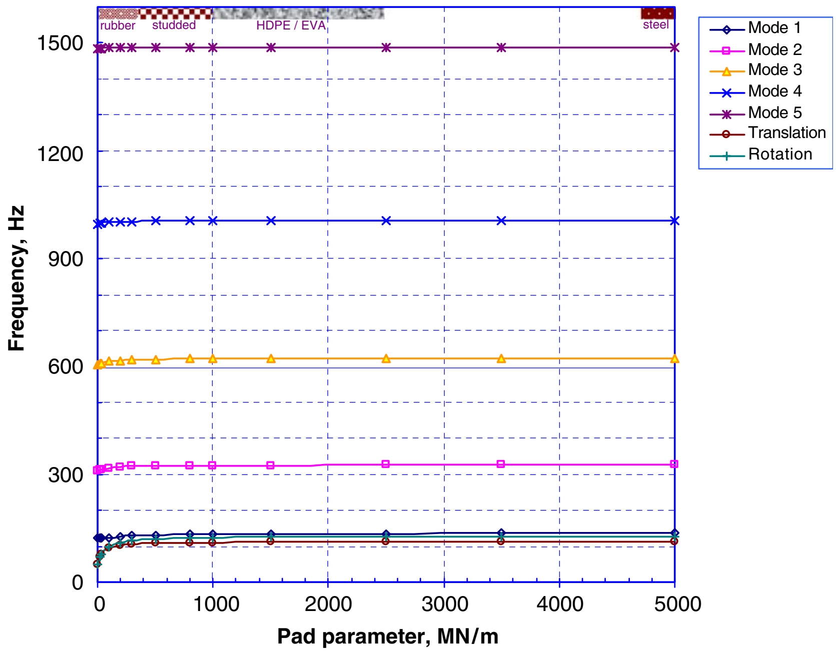


Fig. 2. Influence of rail pad parameters on mode frequencies of an in situ railway concrete sleeper.

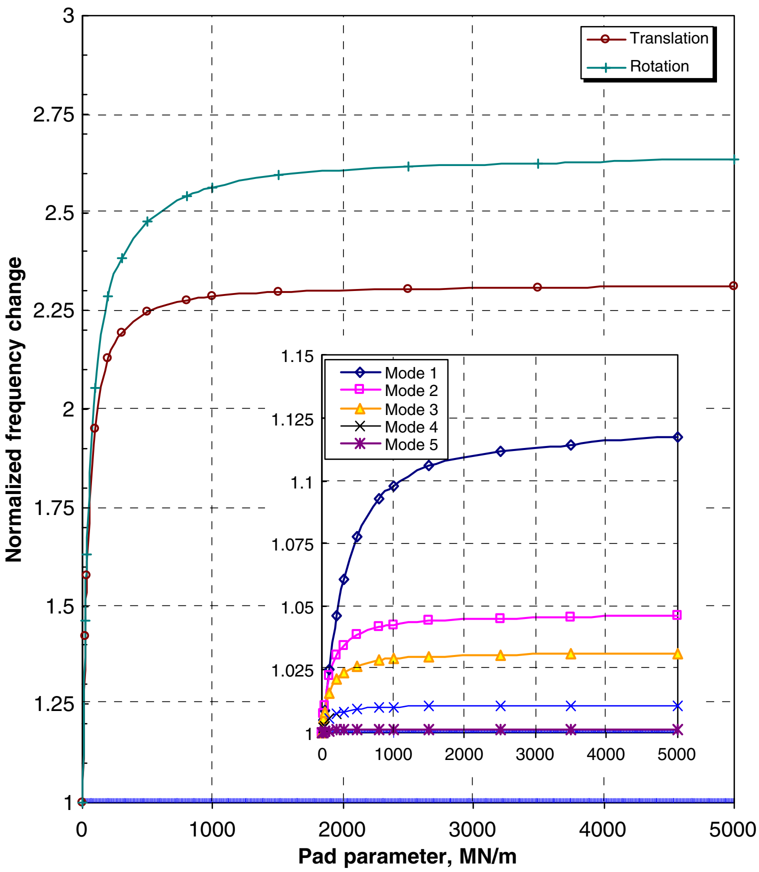


Fig. 3. Normalized modal changes of an in situ railway concrete sleeper.

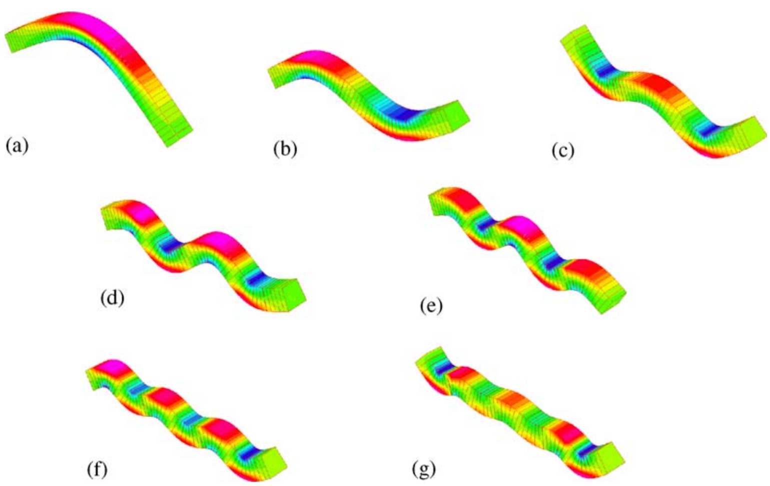


Fig. 4. Mode shapes of an in situ sleeper (when $k_p = \infty$): (a) 1st mode (118.88 Hz), (b) 2nd mode (249.64 Hz), (c) 3rd mode (374.86 Hz), (d) 4th mode (725.26 Hz), (e) 5th mode (1365.44 Hz), (f) 6th mode (2062.78 Hz), and (g) 7th mode (2382.12 Hz).

dynamic mode shapes of which the rigid body motion has been eliminated by the rail-seat constraints and the dominant resonances exist only in transverse vibrations.

4. Conclusions

Free vibration characteristics of the in situ railway concrete sleepers in a track structure system with different types of rail pads are examined using the finite element approach. The two-dimensional simulation based on the Timoshenko beam and spring elements has been verified and found in very good agreements with previous investigations. The results show that the rail pad parameters have the nonlinear effect on the effective stiffness significantly influencing the first three modes of the in situ track system, and can remarkably affect the frequencies and mode shapes of the further modes. At high effective stiffness, the flexural mode shapes are modified due to the fact that the stiff constraints are naturally imposed at the rail seats of the in situ sleeper. Eventually, the translation and rotation modes might be eliminated and in turn the flexural modes take place if the stiffness of spring supports reaches infinity. Of most practical concern, this information indicates the use of modern rail pads or the condition of worn pads that affects the resonant frequencies of the in situ sleepers, which might shift close to typical corrugation-passing frequencies (e.g. 300 or 600 Hz) and so on. Resonance of the second or third dynamic bending modes will expedite incurring cracks on the concrete sleepers. As mentioned, these vibration characteristics of in situ concrete sleepers are very important for the development of dynamic health monitoring tool for in situ concrete sleepers in the modern railway track at different periods: before and after maintenance. This model has been very useful and has led to a study of the effect of ballast contacts on the in situ sleeper dynamic behaviors for which it is inconvenient to solve through the closed-form analytical formulations.

Acknowledgements

The authors are grateful to acknowledge the Cooperative Research Centre (CRC) for Railway Engineering and Technologies, Australia, for the financial support as part of Project #5/23. The authors also would like to thank Dr. Jens Nielsen of Chalmers University of Technology for providing some supporting information.

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